

I.D. #	
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Subject	Phys. 560
Course	Section
Instructor	
Date	

Receiving or giving aid in a final examination is a cause for dismissal from the University.

(Begin Writing on this page)

Lecture 4:

Last time, we showed that the structure function is defined in terms of the radial distribution fcn.

$$S(r) = 1 + \rho \int g(r) e^{ik \cdot r} dr,$$

with $g(r)$ the RDF. $g(r)$ tells you the probability of finding a particle at r given there is one at the origin. See Pages 115-116 for pictures of $g(r)$ in solids, liquids, etc. A few materials of note are as follows:

a.) Liquid Crystals: Smectic ordering breaks both rotational and translational order. Nematic just involves rotational symm. breaking. $\begin{array}{c} O \\ O \\ O \\ O \end{array}$ (S): $\begin{array}{ccccc} O & O & O & O \\ O & O & O & O \end{array}$ (N).

b.) Glasses: cool a liquid quickly. For window glass the atoms get stuck in a glassy phase for a rate of 10 k/s. Ni requires a cool rate of 10⁷ k/s. The density as well as the viscosity increase.

$$\eta \propto e^{\frac{C}{T-T_0}} \text{ exponential growth of the viscosity.}$$

C.) Quasicrystals: Crystals with 3-fold and 5-fold symmetry.

$\text{Al}_{86}\text{Mn}_{14}$ was the first material. The order involves a quasi-periodic tiling. Two periods with the period irrational. $f(x) = \cos x + 80\sin x$. The ratio is $\sqrt{5}$.

1-d tiling.

$$x_n = n + \alpha + \frac{1}{\tau} \lceil \frac{n}{\tau} + \beta \rceil$$

arbitrary real #. largest integer fcn. arbitrary #

$$\tau^2 - \tau - 1 = 0 \Rightarrow \tau = \frac{1 + \sqrt{5}}{2}.$$

$$x_n - x_{n-1} = \begin{cases} 1 & \tau \\ 1 + \tau & \end{cases}$$

So here we have 2 separations. Let's call them L and S. The sequence will be of the Fibonacci type.

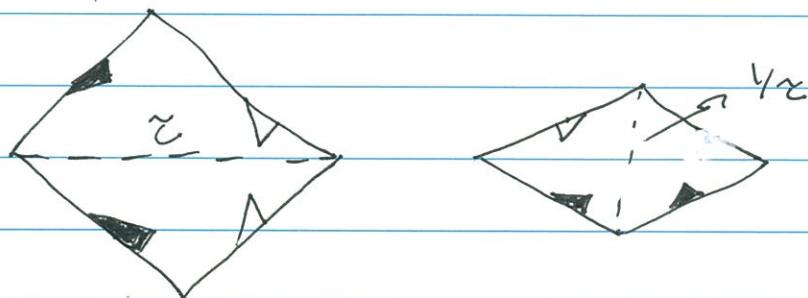
$$x_{n+1} = x_n x_{n-1}$$

$$\text{Let } x_0 = L, x_1 = LS, x_2 = LSL, x_3 = LSLLS, \dots$$

All the S's never touch. The pattern is not periodic.

$$\bullet \quad L \quad S \quad L \quad L \quad S \quad \dots$$

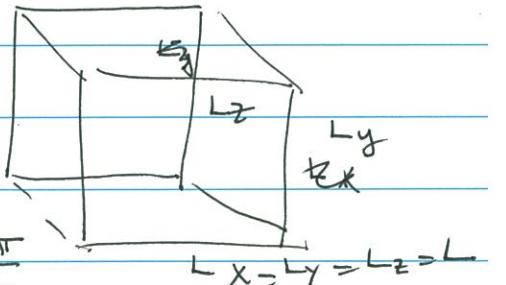
2-D: Penrose tiling



join together only when arrows match.

2.) Free Electrons:

$$H = \sum_i p_i^2 / 2m.$$



$$\vec{k} = \frac{2\pi}{L} (\underbrace{l_x, l_y, l_z}_{}). \Rightarrow \Delta k_i = \frac{2\pi}{L}$$

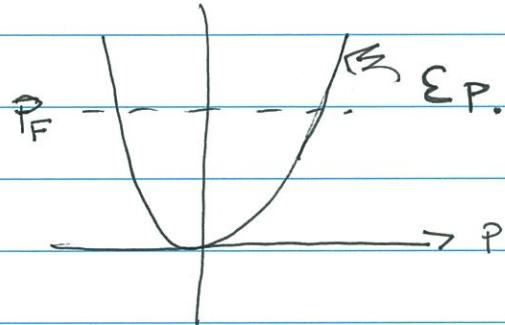
ϵ integers.

$$\epsilon_{k,i} = (k_i)^2 / 2m.$$

$$\psi(\vec{r}) = \psi(\vec{r} + \vec{L}). \Rightarrow \psi_k = e^{i\vec{k}\cdot\vec{r}} / \sqrt{V}$$

$(\frac{2\pi}{L})^3$ = Volume of each k -point.

$$p = \lambda k.$$



A.) Number of Particles

→ highest occupied. Let's call this momentum p_F .

$$N(T=0) = 2 \sum_p \Theta(p_F - p)$$

↑

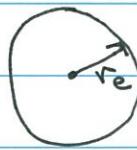
Heaviside step fcn.

$$\sum_p \rightarrow V \int \frac{d^3 p}{(2\pi\lambda)^3}.$$

$$\Rightarrow N(T=0) = \frac{2V}{(2\pi\lambda)^3} 4\pi \int_0^{p_F} p^2 dp$$

$$\Rightarrow n_e = N/V = p_F^3 / 3\pi^2 \lambda^3.$$

(4.)



$$1 = \frac{4\pi}{3} r_e^3 n_e$$

def. of r_e .

$$\Rightarrow r_e^3 = \frac{3}{4\pi} \frac{3\pi^2 k^3}{P_F^3}$$

$$\Rightarrow r_e = 1.92 \lambda / P_F.$$

we can define a dimensionless spacing

$$r_s = \frac{r_e}{a_0} = 1.92 \frac{\hbar}{P_F} \cdot \frac{me^2}{\lambda^2}.$$

What does this # mean? Let's rewrite it
noting that in Q.M. $P \sim \lambda/L \sim \lambda/r_e$.

$$\begin{aligned} r_s &= \frac{r_e}{a_0} = \frac{r_e^2 \cdot me^2}{r_e \lambda^2} \\ &= \frac{e^2 / r_e}{\lambda^2 / m r_e^2} \approx \frac{V_{ee}}{K.E.} \end{aligned}$$

r_s denotes the strength of the potential relative to the kinetic energy. more in class.

	Li	Na	K	Rb	Cs
r_s	3.25	3.93	4.86	5.2	5.62

Cs is the most correlated metal.

Chemical potential: $\mu_0 = P_F^2 / 2m = 3.1 \text{ eV}$ for Na.

Ground-state energy

$$E = 2 \sum_p g_p$$

$$= \frac{2V}{(2\pi\lambda)^3} \frac{4\pi}{10m} \int_0^{P_F} p^4 dp$$

$$= \frac{2V}{(2\pi\lambda)^3} \frac{4\pi}{10m} P_F^5$$

$$\text{Note } V = N \frac{3\pi^2 \lambda^3}{P_F^3}, \mu_0 = P_F^2 / 2m.$$

$$\Rightarrow E = \frac{3}{5} N \mu_0$$

$$\text{Pressure: } P = -\left(\frac{\partial E}{\partial V}\right)_{T,N} = -\frac{2}{\partial V} \left(\frac{3}{5} N \mu_0\right)$$

$$\mu_0 \propto P_F^2 \text{ but } P_F \propto V^{-1/3}.$$

$$\Rightarrow \mu_0 \sim V^{-2/3}$$

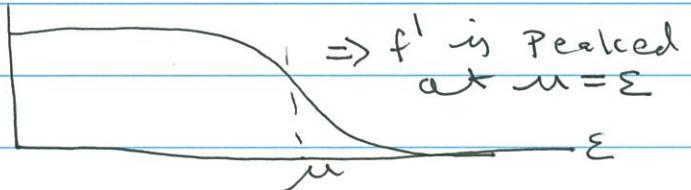
$$\Rightarrow \frac{\partial \mu_0}{\partial V} = -\frac{2}{3} \frac{\mu_0}{V}.$$

$$\Rightarrow P = \frac{2}{5} \frac{N}{V} \mu_0 = \boxed{\frac{2}{5} N e \mu_0}.$$

This pressure is due entirely to Pauli exclusion!

B.) Finite Temperature:

$$f_p(\epsilon) =$$



(6)

$$C_V = T \left(\frac{\partial S}{\partial T} \right)_V \text{ heat capacity.}$$

Let's use the entropy from stat. mech.

$$S = k_B \ln W = -k_B \sum_{p,o} f_{p,o} \ln f_{p,o} + (-f_{p,o}) \ln (-f_{p,o}).$$

$$\Rightarrow SS = -2k_B \sum_p \delta f_p \ln f_p / (1-f_p).$$

$$= 2V k_B \int \frac{d^3 P}{(2\pi\lambda)^3} \delta f_p \left(\frac{\epsilon_p - u}{k_B T} \right).$$

Density of states.

$$N(\epsilon) = 2 \int \frac{d^3 P}{(2\pi\lambda)^3} \delta(\epsilon - \epsilon_p)$$

$$= \frac{2 \cdot 4\pi}{(2\pi\lambda)^3} \int \int \int \frac{P^2}{\epsilon_p^2} \frac{dP}{d\epsilon_p} \delta(\epsilon - \epsilon_p) d\epsilon_p$$

$$N(\epsilon_p) = m P / \pi^2 \lambda^3$$

$$\Rightarrow SS = \frac{V}{T} \int_0^\infty d\epsilon_p \delta f_p N(\epsilon_p) (\epsilon_p - u).$$

$$\text{Let's consider } I = \int_0^\infty d\epsilon f(\epsilon) h(\epsilon).$$

integrate by parts.

$$I = \int_0^\infty \underbrace{f'}_{H(\epsilon)} H(\epsilon) d\epsilon \xrightarrow{\text{Peaked at } (\epsilon=u)} \text{do expansion!}$$

$$H(\epsilon) = - \int_0^\epsilon h(x) dx.$$

$$H(\epsilon) = H(\mu) + (\epsilon - \mu) \frac{\partial H}{\partial \epsilon} \Big|_{\epsilon=\mu} + \frac{1}{2} (\epsilon - \mu)^2 \frac{\partial^2 H}{\partial \epsilon^2} \Big|_{\epsilon=\mu} + \dots$$

Here $H = N(\epsilon)(\epsilon - \mu)$

$$L_j = - \int_0^\infty (\epsilon - \mu)^j f'(\epsilon) d\epsilon$$

$$f' = \beta e^{\beta(\epsilon - \mu)} / (1 + e^{\beta(\epsilon - \mu)})^2.$$

$$x = \beta(\epsilon - \mu) ; dx = \beta d\epsilon.$$

$$\Rightarrow L_j = \frac{1}{\beta^j} \int_0^\infty x^j \frac{e^x}{(e^x + 1)^2}.$$

The integrand is odd \Rightarrow only even j 's survive.

$$L_0 = 1$$

$$L_2 = \frac{\pi^2}{3} (k_B T)^2.$$

\vdots

$$\Rightarrow I = H(\mu) - \frac{L_2 H''(\mu)}{2T} + \dots$$

$$= - \int_0^\mu h(\epsilon) + \frac{\pi^2}{3} (k_B T)^2 h'(\epsilon) .$$

$$\Rightarrow \int_0^\infty d\epsilon S f h(\epsilon) = \frac{\pi^2}{3} h'(\epsilon = \mu) k_B T \delta T.$$

$$h = N(\epsilon)(\epsilon_p - \mu) \Rightarrow h'(\epsilon = \mu) = N(\epsilon_p)$$

$$\Rightarrow \delta S = \frac{\pi^2}{3} k_B N(\epsilon_F) T \delta T.$$

$$\Rightarrow C_V = \frac{\delta S}{\delta T} = V \frac{\pi^2}{3} k_B^2 N(\epsilon_F) T.$$

$$\begin{aligned}\Rightarrow \frac{C_V}{T} &= \frac{\pi^2}{3} k_B^2 \frac{3\pi^2 \chi^3}{P_F^2} \frac{m \beta_F}{\pi^2 \chi^3} T \\ &= \frac{\pi^2}{2} k_B \left(\frac{2m}{P_F^2} \right) k_B T \\ &= \frac{\pi^2}{2} k_B T / T_F.\end{aligned}$$

\Rightarrow only electrons within T of T_F contribute

$$\gamma \equiv \frac{C_V/T}{k_B} = \frac{\pi^2 k_B}{2 T_F} \propto m$$

	Li	Na	K	Rb	Cs	UBe ₁₃
γ	1.65	1.38	2.08	2.63	3.97	1100

γ is the Sommerfeld constant which is an indication of the mass. While, as expected, Cs has the largest value of γ , UBe₁₃ is the GOAT. (Sorry I am watching tennis)
 UBe₁₃ is a heavy-Fermion! Why are the electrons in UBe₁₃ so heavy?
 = Good thesis problem!